Math 3305, Chapter 1, Sections 1.4 and 1.5

Note: There will only be Popper 1.4 in this video and not a Popper 1.5!

Here we'll look at polygons and solids – including looking at the Earth!

We begin with a very technical definition and an understanding that

$P_{n+1} = P1$. This is an important part of the definition and we'll get to what it means in just a moment!

We will take a finite number of segments (n! segments) and join them end to end, *not factorial although it's finite too* looping around to join the end of the last segment to the beginning of the first segment. No crossing lines. Only two segments per intersection...
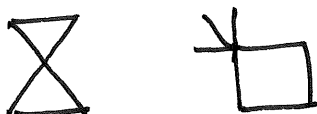
"A polygon is a union of distinct line segments lying in a plane with no three consecutive vertices on a common line and no two sides meeting except at a vertex and only when the sides are in consecutive order".

Note: valuable definition. It is or it isn't. No inbetween. Like logic!

Examples of polygons:
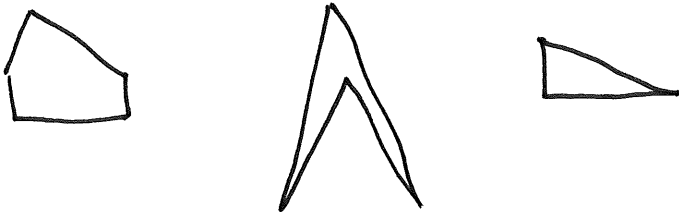
Not a polygon:

Suppose we have 4 vertices. P1, P2, P3, and P4. Each one is a union of two segments in order so that segment 4 links back to segment 1, no crossings. No lining up P1 and P2 to be a different straight line.

In mathier talk: the perimeter $p = \sum_{i=1}^{n} P_1 P_{i+1}$. So in our example we'd have

P1 P2 linked to P2P3 linked to P3P4 and P4P5...EXCEPT that P5 = P1 by our introductory understanding (P_{n+1} = P1). All that does is put the math notation in order!

In the caption on page 19, it is further explained that polygons are closed curves with each side lying on a straight line and that two different sides cannot "cross" each other. Note that a polygon is all in one plane (2D). And the interior of a polygon can be convex. All triangles are convex but polygons with more than three sides need not be.

Popper 1.4, Question One

Which of the following is/are a polygon/s?

I.   III.

II.   IV.
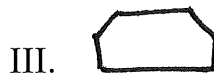
A.   I and III only

B.   II only

C.   II and IV only

D.   All of them

Now polygons can be (carefully) be joined together along the sides to make 3D polyhedrons (3D). Not every polygon can be joined to any other arbitrary polygon to make this solid figure, but if you pick carefully you can do it! Polyhedra is the older plural; of late folks have been saying polyhedrons. Either works.

A cube is a simple polyhedron. Let's look at one that is "exploded". Note the square bases (the top is called a "base" too).
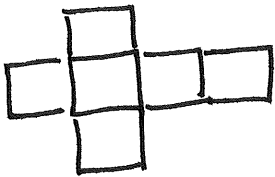
A polyhedron is the surface of a solid that has flat faces made of polygons – page 20. A Bucky ball is a polyhedron and a sphere is not. Again, a definition. What characterizes a definition in math?

Essay 1.4, One

Look up a "Bucky Ball". Get its definition. Write a brief essay about the surface of one. How does the definition of polyhedron help you categorize it? End your essay with a paragraph about why we care about definitions so much in math!

Popper 1.4, Question Two

A polygon is 3D and a polyhedron is 2D.

A.   True

B.   False

More definitions:

A prism is a polyhedron with two faces, called bases, that are congruent and lie in parallel planes. "top" is gone! The other faces called lateral faces are required to be parallelograms. If the lateral faces are a special case of parallelograms, rectangles, the prism is a right prism. Otherwise it is an oblique prism. Let's look at a set diagram of this. Polyhedron, prism, right prism, oblique prism. Great pictures on the net and on page 21.
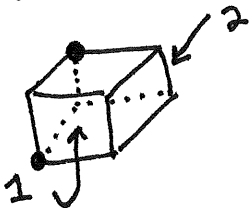
Regular side   Oblique side

Squares are a subset of Rectangles!   *so a cube is a right prism*

Regular polyhedron. A polyhedron is regular, if given any first vertex and face at that first vertex along with any second vertex and face at that second vertex, there is a rotation, taking the first face at the second face at the second vertex.

Say What?

   a rotations

Popper 1.4 Question 3

In a regular polyhedron, the lateral faces are rectangles.

A.   T

B.   F

Ok now all Platonic Solids are regular polyhedrons. And there are only 5 of them. Here's the list:

Tetrahedron	tetra – 4 sides	fire
Cube	6 sides	earth
Octahedron	octa – 8 sides	air
Dodecahedron	dodeca – 12 sides	water
Icosahedron	icosa – 20 sides	the universe

See the pictures pages 23 and 24.

1.4 Essay Two

Look up a Moravian Star. Using the more modern definition (28 spikes), discuss whether or not it meets the definition of a polyhedron. This is more of an opinion piece. I'll be looking at your arguments more than I'll be grading it "right" or "wrong" based on your opinion. How well you **defend** your opinion matters most!

So spheres are not polyhedras. They can be solids, though, if you include the surface and the interior points. And the surface area of any sphere can have Spherical geometry on it! We need to discuss points and lines of spherical geometry. The points have the formula
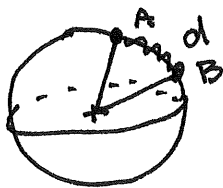
$$x^2 + y^2 + z^2 = 1$$

when using the unit sphere centered at (0, 0, 0). The formula in the text is for ANY sphere centered anywhere.

Lines are the intersection of a plane through the center and the surface of the sphere.
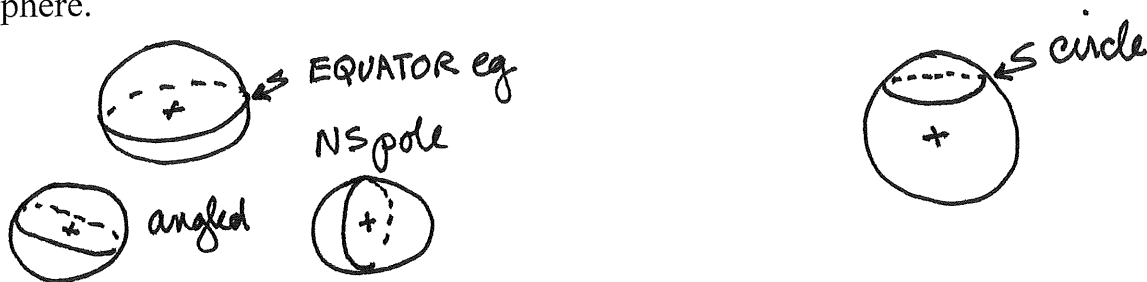
Let's look at a cross section and look at distance on the unit sphere again.

Popper 1.4, Question 4

A plane through the surface of a sphere that does NOT contain the center of the sphere creates a circle, not a line (Great Circle).

A.   T

B.   F

Now, while we're on spheres. Let's do 1.5 and then finish up with the Euler number (which is the end of 1.4).

Section 1.5

That is Earth is roundish was known to Euclid and a pretty good measurement was calculated by Eratosthenes long before Christ was born. The knowledge was lost to Europe because of the burning of the libraries at Constantinople. It was rediscovered centuries later by the Europeans.

Let's look at a cross section of a sphere again. Can you see that the shortest distance between two points of the surface is a great circle arc? If not, grab a ball and put on the equator, which is a great circle. Hold the ball in one hand and "walk" your fingers straddling the equator. Do you see that your fingers move the same distance on either side of the equator? Move up from the equator and walk in a circle but closer to the North Pole, ie on a circle and not a line. A plane though this circle does not contain the center of the sphere, remember? Do you see that the finger closer to the equator is talking longer steps than the one closer to the North Pole? This is just like walking down the middle of a straight corridor on Earth or walking in a small circle. And the "walking" on the sphere like that is the origin of calling great circles "lines" and latitude lines "circles".

1.4 Essay Three

Read the caption for Figure 3. Write a brief essay about how Eratosthenes did his measurements (it's really pretty clever!) and look up the Earth's actual circumference at the equator. How close was he?

Technically the Earth is an oblate spheroid and the "circumference" north pole to south pole is slightly different than the "circumference" around the equator.

Ok back to 1.4 with the Euler Number. Polyhedrons have faces, edges, and vertices. And interestingly when you take the Platonic Solids, as different from one another as they are, the Euler Number for each is the same: two.

You get the Euler Number by counting the faces (F), vertices (V), and edges (E) and filling in the formula:

$F + V - E$

It turns out that because the Platonic Solids are all "topologically equivalent" to the sphere, they all have the same Euler number. Now Topology is often called Rubber Geometry because it stretches and pulls surfaces and shapes with functions to make new surfaces and shapes. Imagine that an icosahedron were made of modeling clay. With some smoothing and pushing you could turn it into a sphere. It's a solid object; you could just smooth out the faces, vertices, and edges with a sculpting tool or your fingers.

BUT new classes of Euler numbers have been discovered since his time (almost the whole century of the 1700s; he was Swiss). The determining property for the Euler number classes is the number of **holes** through the object. Like a torus or doughnut is different from the Platonic solids because there's one hole…and topology can't "fix" that. And imagine two holes or three!

Popper 1.4, Question Five

The Euler Number of a tetrahedron is 2.

A.   T

B.   F

1.4 Essay Four

Look up Euler number. Find out the Euler number for a torus and an object with two holes. Write a brief essay about the history and properties of objects with Euler numbers.

Ok. That's it for Chapter 1

The essays and the popper will finish Sections 1.4 and 1.5

NOW. Also in the list of Chapter 1 Homework are the following review problems from page 33

#2,

#8

1.4 Ms. Leigh One

Find the distance from (1, 3) to (6, 10) in exact square root form and then find a nearby rational mixed number using our new technique.

1.4 Ms. Leigh Two

Given a pentagonal pyramid, predict the Euler number. Find the number of faces, edges, and vertices and calculate the Euler number; show your work and an exploded sketch on this. Cutting out one and putting it together one will help. See page 22 for vocabulary help on pyramids.

Turn in ALL the homework in one document under the assignments tab.

See the calendar for due dates.